SURDS

① $(6 - \sqrt{8})^2$

$\sqrt{8} = \sqrt{4 \times 2} = \sqrt{4} \times \sqrt{2} = 2\sqrt{2}$

$= (6 - 2\sqrt{2})(6 - 2\sqrt{2})$

FOIL $= 36 - 12\sqrt{2} - 12\sqrt{2} + 4(\sqrt{2})^2$

$= 36 - 24\sqrt{2} + 8$

$= 44 - 24\sqrt{2}.$

② $\dfrac{\sqrt{3} + \sqrt{27}}{\sqrt{2}}$

$\sqrt{27} = \sqrt{9 \times 3} = \sqrt{9} \times \sqrt{3} = 3\sqrt{3}$

$= \dfrac{\sqrt{3} + 3\sqrt{3}}{\sqrt{2}} = \dfrac{4\sqrt{3}}{\sqrt{2}} \times \dfrac{\sqrt{2}}{\sqrt{2}} = \dfrac{4\sqrt{6}}{2} = 2\sqrt{6}$

$\sqrt{2} \times \sqrt{2} = \sqrt{4} = 2$

Now write $2\sqrt{6}$ inside a single root - ie in the form $\sqrt{k}$

$2 = \sqrt{4}$

$\Rightarrow 2\sqrt{6} = \sqrt{4} \times \sqrt{6} = \sqrt{4 \times 6} = \sqrt{24}.$

③ $\sqrt{48} + \sqrt{108} = k\sqrt{6}$

$\sqrt{8\times6} + \sqrt{18\times6} = \sqrt{8}\times\sqrt{6} + \sqrt{18}\times\sqrt{6}$

$\sqrt{8} = \sqrt{4\times2} = \sqrt{4}\times\sqrt{2} = 2\sqrt{2}$

$\sqrt{18} = \sqrt{9\times2} = \sqrt{9}\times\sqrt{2} = 3\sqrt{2}$

$= 2\sqrt{2}\times\sqrt{6} + 3\sqrt{2}\times\sqrt{6}$

$= (2\sqrt{2} + 3\sqrt{2})\times\sqrt{6}$

$= 5\sqrt{2}\times\sqrt{6}.$

④ a) $(3 + 2\sqrt{2})(4 - \sqrt{2})$

$(\sqrt{2})^2 = \sqrt{2}\times\sqrt{2} = \sqrt{4} = 2$

FOIL $= 12 - 3\sqrt{2} + 8\sqrt{2} - 2(\sqrt{2})^2$

$= 12 + 5\sqrt{2} - 4$

$= 8 + 5\sqrt{2}.$

b) $\left(\frac{10 + 3\sqrt{2}}{\sqrt{2}}\right) \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{10\sqrt{2} + 3(\sqrt{2})^2}{(\sqrt{2})^2} = \frac{10\sqrt{2} + 3\times2}{2}$

$= \frac{10\sqrt{2} + 6}{2} = 5\sqrt{2} + 3$

(5) $(3+\sqrt{a})(4+\sqrt{a}) = 17 + k\sqrt{a}$

FOIL $12 + 3\sqrt{a} + 4\sqrt{a} + (\sqrt{a})^2 = 17 + k\sqrt{a}$

$12 + 7\sqrt{a} + a = 17 + k\sqrt{a}$

$12 + a + 7\sqrt{a} = 17 + k\sqrt{a}$

whole No.s : whole No.s $\Rightarrow$ $12 + a = 17 = \underline{a = 5}$

$\sqrt{a}$'s : $\sqrt{a}$'s $\Rightarrow$ $7\sqrt{a} : k\sqrt{a} \Rightarrow \underline{k = 7}$

6a) $(5 + 3\sqrt{2})^2 = (5 + 3\sqrt{2})(5 + 3\sqrt{2})$

FOIL $= 25 + 15\sqrt{2} + 15\sqrt{2} + 9(\sqrt{2})^2$

$= 25 + 30\sqrt{2} + 18$

$= 43 + 30\sqrt{2}$.

b) Using Answer from a) $(5 + 3\sqrt{2})^2 = \cancel{43} + 30\sqrt{2} = \cancel{p} + \frac{q}{\sqrt{8}}$

whole No. : whole No. $\Rightarrow p = 43$

Root : Root $\Rightarrow$ $30\sqrt{2} = \frac{q}{\sqrt{8}}$ $\Rightarrow 30\sqrt{2} \times \sqrt{8} = q$

$\Rightarrow 30\sqrt{16} = q$

$\Rightarrow 30 \times 4 = q$

$\Rightarrow \underline{120 = q}$

(7) a) $(5 - \sqrt{8})(7 + \sqrt{2})$

$\sqrt{8} = \sqrt{4 \times 2} = \sqrt{4} \times \sqrt{2} = 2\sqrt{2}$

$= (5 - 2\sqrt{2})(7 + \sqrt{2})$

FOIL $= 35 + 5\sqrt{2} - 14\sqrt{2} - 2(\sqrt{2})^2$

$= 35 - 9\sqrt{2} - 4 = \underline{31 - 9\sqrt{2}}$

b) $\left(\frac{3c - \sqrt{c}}{\sqrt{c}}\right) \times \frac{\sqrt{c}}{\sqrt{c}} = \frac{3c\sqrt{c} - (\sqrt{c})^2}{(\sqrt{c})^2}$

$= \frac{3c\sqrt{c} - c}{c} = 3\sqrt{c} - 1$

(8) (see Q5)

$(5 - \sqrt{x})(5 - \sqrt{x}) = 7 - 20\sqrt{2}$

FOIL $25 - 5\sqrt{x} - 5\sqrt{x} + (\sqrt{x})^2 = 7 - 20\sqrt{2}$

$25 + x - 10\sqrt{x} = 7 - 20\sqrt{2}$

whole = whole $\Rightarrow$ $25 + x = y$

rock = rock $\Rightarrow$ $10\sqrt{x} = 20\sqrt{2} \Rightarrow \sqrt{x} = 2\sqrt{2}$

$= \sqrt{4} \times \sqrt{2}$

$= \sqrt{8}$

$\Rightarrow x = 8$

$\Rightarrow y = 33.$

(9) See Q5, Q8...

$$(1+\sqrt{x})(3+\sqrt{x}) = 7+4\sqrt{5}$$

FOIL $\quad 3+\sqrt{x}+3\sqrt{x}+(\sqrt{x})^2 = 7+4\sqrt{5}$

$$3+x+4\sqrt{x} = 7+4\sqrt{5}$$

root = root $\Rightarrow \quad 4\sqrt{x} = 4\sqrt{5} \Rightarrow \underline{x=5}$

whole : whole $\Rightarrow \quad 3+x = y \Rightarrow \underline{y=8}$

(10) See Q5, Q8, Q9

$$\left(\sqrt{a}+\sqrt{8a}\right)^2 = 54+6\sqrt{2}$$

$\sqrt{8} = \sqrt{4}\times\sqrt{2} = 2\sqrt{2}$

$$\left(\sqrt{a}+\sqrt{8}\times\sqrt{a}\right)^2 = 54+6\sqrt{2}$$

$$\left(\sqrt{a}+2\sqrt{2}\sqrt{a}\right)^2 = 54+6\sqrt{2}$$

FOIL $\quad (\sqrt{a}+2\sqrt{2}\sqrt{a})(\sqrt{a}+2\sqrt{2}\sqrt{a}) = 54+6\sqrt{2}$

$$(\sqrt{a})^2+2\sqrt{2}(\sqrt{a})^2+2\sqrt{2}(\sqrt{a})^2+(2\sqrt{2})^2(\sqrt{a})^2 = 54+6\sqrt{2}$$

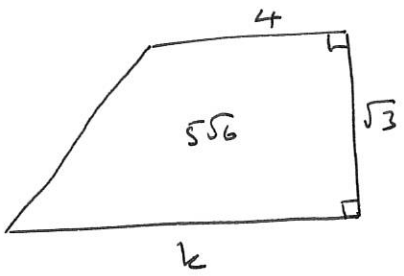
$$a+2\sqrt{2}a+2\sqrt{2}a+8a = 54+6\sqrt{2}$$

$$9a+4a\times\sqrt{2} = 54+6\sqrt{2}$$

whole = whole $\quad 9a = 54 \Rightarrow a = 6$

root = root $\quad 4a = b \Rightarrow b = 24$

(11)



See F.S. for area of a Trapezium.

$$A = \frac{1}{2}(a+b)h = \frac{1}{2}(k+4)\sqrt{3} = 5\sqrt{6}$$

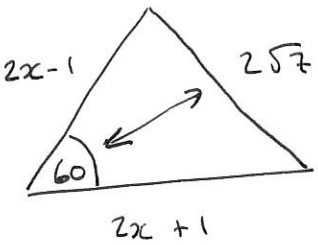
$$\Rightarrow (k+4)\sqrt{3} = 10\sqrt{6}$$

$$\Rightarrow k+4 = \frac{10\sqrt{6}}{\sqrt{3}} = 10\sqrt{2}$$

use calc.

$$\Rightarrow \underline{k = 10\sqrt{2} - 4}$$

(12)



Cosine Rule:

$$(2\sqrt{7})^2 = (2x-1)^2 + (2x+1)^2 - 2(2x-1)(2x+1)\times \underline{\cos 60}$$

use calc

use calc $(2x-1)(2x-1)$ $(2x+1)(2x+1)$

$$\Rightarrow 28 = (4x^2 - 2x - 2x + 1) + (4x^2 + 2x + 2x + 1) - 2\left(4x^2 + \cancel{2x} - \cancel{2x} - 1\right)\times\frac{1}{2}$$

$$\Rightarrow 28 = 4x^2 - \cancel{4x} + 1 + \cancel{4x^2} + \cancel{4x} + 1 - \cancel{4x^2} + 1$$

$$\Rightarrow 28 = 4x^2 + 3$$

$$\Rightarrow 4x^2 - 25 = 0$$

Quadratic formula $a = 4$ $b = 0$ $c = -25$

$$x = \frac{0 \pm \sqrt{(0)^2 - 4(4)(-25)}}{8} \qquad x = \frac{5}{2} \text{ or } -\frac{5}{2}.$$

↑ Not possible

$$x = \frac{5}{2} = 2.5.$$