

## Argand Diagrams 1: Modulus and Argument

The modulus of a complex number  $z = a + bi$  is  $|z| = \sqrt{a^2 + b^2}$ .

The argument of a complex number,  $\arg z$ , is the angle between the positive real axis and the line joining  $z$  to the origin in the Argand diagram, measured anticlockwise.

$\arg z = \arctan \frac{b}{a}$  in the first quadrant

$\arg z = \pi - \arctan \frac{b}{a}$  in the second quadrant

$\arg z = -(\pi - \arctan \frac{b}{a})$  in the third quadrant

$\arg z = -\arctan \frac{b}{a}$  in the fourth quadrant

## Examples

1. Find the modulus and argument of  $z = 2 + 7i$
  
  
  
  
  
  
  
  
  
  
2. Find the modulus and argument of  $z = -4 - i$

For a complex number  $z$  with  $|z| = r$  and  $\arg z = \theta$ , the modulus-argument form is  $z = r(\cos \theta + i \sin \theta)$ .

## Examples

1. Express  $z = -\sqrt{3} + i$  in the form  $r(\cos \theta + i \sin \theta)$ , where  $-\pi < \theta \leq \pi$
2. Express  $z = -1 - i$  in the form  $r(\cos \theta + i \sin \theta)$ , where  $-\pi < \theta \leq \pi$

For any two complex numbers  $z_1$  and  $z_2$

$$|z_1 z_2| = |z_1| |z_2|$$

$$\arg(z_1 z_2) = \arg z_1 + \arg z_2$$

$$\left| \frac{z_1}{z_2} \right| = \frac{|z_1|}{|z_2|}$$

$$\arg \left( \frac{z_1}{z_2} \right) = \arg z_1 - \arg z_2$$

## Examples

1.  $z_1 = 2 \left( \cos \frac{\pi}{15} + i \sin \frac{\pi}{15} \right)$  and  $z_2 = 2 \left( \cos \frac{2\pi}{5} + i \sin \frac{2\pi}{5} \right)$   
Express  $z_1 z_2$  in the form  $x + iy$

2. Express  $\frac{\sqrt{2} \left( \cos \frac{\pi}{12} + i \sin \frac{\pi}{12} \right)}{2 \left( \cos \frac{5\pi}{6} + i \sin \frac{5\pi}{6} \right)}$  in the form  $x + iy$