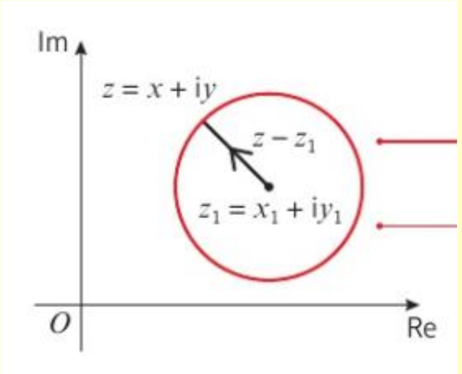
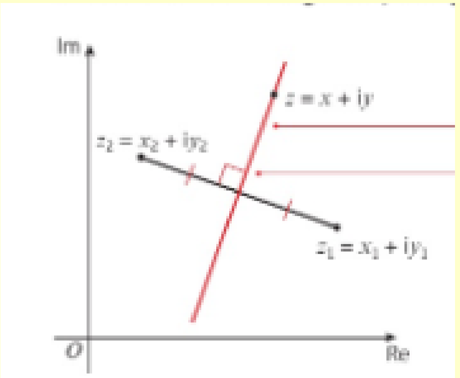
Argand Diagrams 2: Loci and Regions

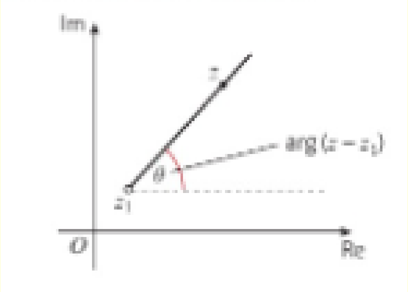
$|z - z_1| = r$ is a circle with centre (x_1, y_1) and radius r.



$|z - z_1| = |z - z_2|$ is the perpendicular bisector of z_1 and z_2.



$\arg(z - z_1) = \theta$ is a half line from the point z_1 making an angle of θ.



Example 1

Given that the complex number $z = x + iy$ satisfies the equation $|z - 12 - 5i| = 3$, find the minimum and maximum values of $|z|$.

Example 2

Given that $|z - 3| = |z + i|$

a. Sketch the locus of z and find the Cartesian equation of this locus.

b. Find the least possible value of $|z|$.

Example 3

Given that $\arg(z+3+2i) = \frac{3\pi}{4}$

a. Sketch the locus of z on an Argand diagram

b. Find the Cartesian equation of the locus

c. Find the complex number z that satisfies both $|z+3+2i| = 10$ and $\arg(z+3+2i) = \frac{3\pi}{4}$

Example 4

a. On separate Argand diagrams, shade in the regions represented by:

i. $|z - 4 - 2i| \leq 2$

ii. $|z - 4| < |z - 6|$

iii. $0 \leq \arg(z - 2 - 2i) \leq \frac{\pi}{4}$.

b. Hence, on the same Argand diagram, shade the region that satisfies

$$\{z \in \mathbb{C}: |z - 4 - 2i| \leq 2\} \cap \{z \in \mathbb{C}: |z - 4| < |z - 6|\} \cap \left\{z \in \mathbb{C}: 0 \leq \arg(z - 2 - 2i) \leq \frac{\pi}{4}\right\}$$