

### Arithmetic Series

1. An arithmetic series has five terms. The first term is 2 and the last term is 32. Find the sum of the series.

$$\begin{array}{ccccccc} 2 & & & & & & 32 \\ & \nearrow & & \nearrow & & \nearrow & \\ & +d & & +d & & +d & \end{array} \quad 4d = 30 \quad d = 7.5$$

(Total 4 marks)

$$S_5 = \frac{5}{2} (2 + 32) = 85$$

2. In an arithmetic sequence, the first term is 5 and the fourth term is 40. Find the second term.

$$\begin{array}{ccccccc} 5 & & & & & & 40 \\ & \nearrow & & \nearrow & & \nearrow & \\ & +d & & +d & & +d & \end{array} \quad 3d = 35 \Rightarrow d = 11 \frac{2}{3}$$

$$\Rightarrow u_2 = 16 \frac{2}{3}$$

(Total 4 marks)

3. Each day a runner trains for a 10 km race. On the first day she runs 1000 m, and then increases the distance by 250 m on each subsequent day.  $u_1 = 1000 \quad d = 250$

- (a) On which day does she run a distance of 10 km in training?

$$u_n = 10000 \Rightarrow 1000 + (n-1)250 = 10000$$

$$\Rightarrow n = 37$$

- (b) What is the total distance she will have run in training by the end of that day? Give your answer exactly.

$$S_{37} = \frac{37}{2} (1000 + 10000) = 203.5 \text{ km}$$

(Total 4 marks)

4. The first three terms of an arithmetic sequence are 7, 9.5, 12.  $u_1 = 7 \quad d = 2.5$

- (a) What is the 41<sup>st</sup> term of the sequence?  $u_{41} = u_1 + 40d = 7 + 40 \times 2.5 = 107$

- (b) What is the sum of the first 101 terms of the sequence?

$$S_n = \frac{n}{2} (2u_1 + (n-1)d) \Rightarrow S_{101} = \frac{101}{2} (14 + 100 \times 2.5) = 13332$$

(Total 4 marks)

5. In an arithmetic sequence, the first term is -2, the fourth term is 16, and the  $n^{\text{th}}$  term is 11 998.

- (a) Find the common difference  $d$ .

$$\begin{array}{ccccccc} -2 & & & & & & 16 \\ & \nearrow & & \nearrow & & \nearrow & \\ & +d & & +d & & +d & \end{array} \quad 3d = 18 \quad d = 6$$

- (b) Find the value of  $n$ .

$$u_n = u_1 + (n-1)d \Rightarrow -2 + (n-1)6 = 11998 \Rightarrow n = 2001$$

(Total 6 marks)

6. Arturo goes swimming every week. He swims 200 metres in the first week. Each week he swims 30 metres more than the previous week. He continues for one year (52 weeks).  $u_1 = 200$

- (a) How far does Arturo swim in the final week?

$$d = 30$$

$$u_{52} = 200 + (52-1) \times 30 = 1730$$

- (b) How far does he swim altogether?

$$S_{52} = \frac{52}{2} (200 + 1730) = 50180 \text{ m}$$

(Total 6 marks)

7. A theatre has 20 rows of seats. There are 15 seats in the first row, 17 seats in the second row, and each successive row of seats has two more seats in it than the previous row.  $u_1 = 15$   $d = 2$

(a) Calculate the number of seats in the 20<sup>th</sup> row.  $u_{20} = 15 + (20-1) \times 2 = 53$

(b) Calculate the **total** number of seats.  $S_{20} = \frac{20}{2} (15 + 53) = 680$

(Total 6 marks)

8. Let  $S_n$  be the sum of the first  $n$  terms of an arithmetic sequence, whose first three terms are  $u_1$ ,  $u_2$  and  $u_3$ . It is known that  $S_1 = 7$ , and  $S_2 = 18$ .

$$\begin{array}{cccc} u_1 & & u_1 + d & & u_1 + 2d \dots \\ \hline 7 & & & & \\ \hline & & 18 & & \end{array}$$

(a) Write down  $u_1$ .  $7$

(b) Calculate the common difference of the sequence.  $7 + (7+d) = 18 \Rightarrow d = 4$

(c) Calculate  $u_4$ .  $7 + 3 \times 4 = 19$

(Total 6 marks)