

Exponentials & Logarithms

1. If $\log_a 2 = x$ and $\log_a 5 = y$, find in terms of x and y , expressions for

(a) $\log_2 5 = \frac{\log_a 5}{\log_a 2} = \frac{y}{x}$

(b) $\log_a 20$.

$\hookrightarrow = \log_a (5 \times 2 \times 2) = \log_a (5) + \log_a (2) + \log_a (2) = y + 2x$ (Total 4 marks)

2. Let $\log_{10} P = x$, $\log_{10} Q = y$ and $\log_{10} R = z$. Express $\log_{10} \left(\frac{P}{QR^3} \right)^2$ in terms of x , y and z .

$2 \log_{10} \left(\frac{P}{QR^3} \right) = 2 [\log_{10} P - \log_{10} Q - 3 \log_{10} R] = 2x - 2y - 6z$ (Total 4 marks)

3. Solve the equation $\log_9 81 + \log_9 \frac{1}{9} + \log_9 3 = \log_9 x$.

$\log_9 (81) + \log_9 \left(\frac{1}{9} \right) + \log_9 (3) = \log_9 \left(81 \times \frac{1}{9} \times 3 \right) = \log_9 (27)$
 $x = 27$. (Total 4 marks)

4. Given that $\log_5 x = y$, express each of the following in terms of y .

(a) $\log_5 x^2 = 2 \log_5 (x) = 2y$

(b) $\log_5 \left(\frac{1}{x} \right) = \log_5 (x^{-1}) = -\log_5 (x) = -y$

(c) $\log_{25} x = \frac{\log_5 (x)}{\log_5 (25)} = \frac{y}{2}$ (Total 6 marks)

5. Solve the equation $\log_{27} x = 1 - \log_{27} (x - 0.4)$. $\Rightarrow \log_{27} (x) + \log_{27} (x - 0.4) = 1$

$\Rightarrow \log_{27} [x(x - 0.4)] = 1 \Rightarrow x^2 - 0.4x = 27 \Rightarrow x^2 - 0.4x - 27 = 0$ (Total 6 marks)

6. Let $a = \log x$, $b = \log y$, and $c = \log z$.

$\Rightarrow x = 5.4$ or -5.4 ignore
 $x = 5.4$

Write $\log \left(\frac{x^2 \sqrt{y}}{z^3} \right)$ in terms of a , b and c . $= \log (x^2) + \log (y^{1/2}) - \log (z^3)$

$= 2 \log (x) + \frac{1}{2} \log (y) - 3 \log (z) = 2a + \frac{1}{2}b - 3c$ (Total 6 marks)

7. (a) Given that $\log_3 x - \log_3 (x - 5) = \log_3 A$, express A in terms of x .

$\log_3 \left(\frac{x}{x-5} \right) = \log_3 (A) \quad A = \frac{x}{x-5}$

(b) Hence or otherwise, solve the equation $\log_3 x - \log_3 (x - 5) = 1$.

$\log_3 \left(\frac{x}{x-5} \right) = 1$ (Total 6 marks)

$\Rightarrow \frac{x}{x-5} = 3^1 \Rightarrow x = 3(x-5)$

$x = 3x - 15$

$15 = 2x$

$7.5 = x$

8. Solve the equation $4^{3x-1} = 1.5625 \times 10^{-2}$.

$$\text{Since } a^x = b \Leftrightarrow \log_a b = x$$

(Total 6 marks)

$$\begin{aligned} \Rightarrow \log_4 (1.5625 \times 10^{-2}) &= 3x - 1 \\ -3 &= 3x - 1 \\ -2 &= 3x \\ -2/3 &= x \end{aligned}$$

9. Non Calculator: Find the **exact** solution of the equation $9^{2x} = 27^{(1-x)}$

$$\text{Common Base} = 3 \quad \therefore \quad (3^2)^{2x} = (3^3)^{1-x}$$

(Total 4 marks)

$$3^{4x} = 3^{3-3x}$$

$$4x = 3 - 3x$$

$$7x = 3 \quad x = 3/7$$

10. Non Calculator: Solve the equation $9^{x-1} = \left(\frac{1}{3}\right)^{2x}$.

$$\text{Common Base} = 3 \quad \therefore \quad (3^2)^{x-1} = (3^{-1})^{2x}$$

(Total 4 marks)

$$3^{2x-2} = 3^{-2x}$$

$$2x - 2 = -2x$$

$$4x = 2$$

$$x = 1/2$$