

Polar Coordinates 2: Area and Tangents

$$\text{Area} = \frac{1}{2} \int_{\alpha}^{\beta} r^2 d\theta$$

Example

Find the area enclosed by the cardioid with equation $r = a(1 + \cos \theta)$

Example

Find the area of one loop of the curve with polar equation $r = a \sin 4\theta$

Example

- a. On the same diagram, sketch the curves with equations $r = 2 + \cos \theta$ and $r = 5 \cos \theta$
- b. Find the polar coordinates of the points of intersection of these two curves
- c. Find the exact area of the region which lies within both curves

To find a tangent parallel to the initial line set $\frac{dy}{d\theta} = 0$

To find a tangent perpendicular to the initial line set $\frac{dx}{d\theta} = 0$

Example

Find the coordinates of the points on $r = a(1 + \cos \theta)$ where the tangents are parallel to the initial line $\theta = 0$

Example

Find the equations and the points of contact of the tangents to the curve $r = a \sin 2\theta$ $0 \leq \theta \leq \frac{\pi}{2}$ that are

- a. Parallel to the initial line
- b. Perpendicular to the initial line

Example

The curve C has equation $r = (p + q \cos \theta)$ where p and q are positive constants and $p > q$.

Prove that the curve is convex for $p \geq 2q$ and has a dimple for $p < 2q$