The Normal Distribution Questions

Q1.

The heights of an adult female population are normally distributed with mean 162 cm and standard deviation 7.5 cm.

(a) Find the probability that a randomly chosen adult female is taller than 150 cm.

Sarah is a young girl. She visits her doctor and is told that she is at the 60th percentile for height.

(b) Assuming that Sarah remains at the 60th percentile, estimate her height as an adult.

The heights of an adult male population are normally distributed with standard deviation 9.0 cm.

Given that 90% of adult males are taller than the mean height of adult females,

(c) find the mean height of an adult male.

(Total 10 marks)

Q2.

The random variable $X \sim N(\mu, 5^2)$ and $P(X < 23) = 0.9192$

(a) Find the value of $\mu$.

(b) Write down the value of $P(\mu < X < 23)$.

(Total 5 marks)

Q3.

Past records show that the times, in seconds, taken to run 100 m by children at a school can be modelled by a normal distribution with a mean of 16.12 and a standard deviation of 1.60

A child from the school is selected at random.

(a) Find the probability that this child runs 100 m in less than 15 s.

On sports day the school awards certificates to the fastest 30% of the children in the 100 m race.

(b) Estimate, to 2 decimal places, the slowest time taken to run 100 m for which a child will be awarded a certificate.
Q4.

The random variable $X$ has a normal distribution with mean 30 and standard deviation 5.

(a) Find $P(X < 39)$. (2)

(b) Find the value of $d$ such that $P(X < d) = 0.1151$ (4)

(c) Find the value of $e$ such that $P(X > e) = 0.1151$ (2)

(d) Find $P(d < X < e)$. (2)

(Total 10 marks)

Q5.

The weight, $X$ grams, of soup put in a tin by machine $A$ is normally distributed with a mean of 160 g and a standard deviation of 5 g. A tin is selected at random.

(a) Find the probability that this tin contains more than 168 g. (3)

The weight stated on the tin is $w$ grams.

(b) Find $w$ such that $P(X < w) = 0.01$ (3)

The weight, $Y$ grams, of soup put into a carton by machine $B$ is normally distributed with mean $\mu$ grams and standard deviation $\sigma$ grams.

(c) Given that $P(Y < 160) = 0.99$ and $P(Y > 152) = 0.90$ find the value of $\mu$ and the value of $\sigma$. (6)

(Total 12 marks)
Q6.

The heights of a population of women are normally distributed with mean $\mu$ cm and standard deviation $\sigma$ cm. It is known that 30% of the women are taller than 172 cm and 5% are shorter than 154 cm.

(a) Sketch a diagram to show the distribution of heights represented by this information.

(b) Show that $\mu = 154 + 1.6449\sigma$.

(c) Obtain a second equation and hence find the value of $\mu$ and the value of $\sigma$.

A woman is chosen at random from the population.

(d) Find the probability that she is taller than 160 cm.

(Total 13 marks)

Q7.

The lifetimes of bulbs used in a lamp are normally distributed.

A company $X$ sells bulbs with a mean lifetime of 850 hours and a standard deviation of 50 hours.

(a) Find the probability of a bulb, from company $X$, having a lifetime of less than 830 hours.

(b) In a box of 500 bulbs, from company $X$, find the expected number having a lifetime of less than 830 hours.

A rival company $Y$ sells bulbs with a mean lifetime of 860 hours and 20% of these bulbs have a lifetime of less than 818 hours.

(c) Find the standard deviation of the lifetimes of bulbs from company $Y$.

Both companies sell the bulbs for the same price.

(d) State which company you would recommend. Give reasons for your answer.

(Total 11 marks)
Q8.

The length of time, \( L \) hours, that a phone will work before it needs charging is normally distributed with a mean of 100 hours and a standard deviation of 15 hours.

(a) Find \( P(L > 127) \).

(b) Find the value of \( d \) such that \( P(L < d) = 0.10 \)

Alice is about to go on a 6 hour journey. Given that it is 127 hours since Alice last charged her phone,

(c) find the probability that her phone will not need charging before her journey is completed.

(Total 10 marks)

Q9.

The weight, in grams, of beans in a tin is normally distributed with mean \( \mu \) and standard deviation 7.8

Given that 10% of tins contain less than 200 g, find

(a) the value of \( \mu \)

(b) the percentage of tins that contain more than 225 g of beans.

The machine settings are adjusted so that the weight, in grams, of beans in a tin is normally distributed with mean 205 and standard deviation \( \sigma \).

(c) Given that 98% of tins contain between 200 g and 210 g find the value of \( \sigma \).

(Total 10 marks)

Q10.

The heights of adult females are normally distributed with mean 160 cm and standard deviation 8 cm.

(a) Find the probability that a randomly selected adult female has a height greater than 170 cm.

Any adult female whose height is greater than 170 cm is defined as tall.

An adult female is chosen at random. Given that she is tall,

(b) find the probability that she has a height greater than 180 cm.
Half of tall adult females have a height greater than $h$ cm.

(c) Find the value of $h$.

(Total 12 marks)

## Mark Scheme

### Q1.

<table>
<thead>
<tr>
<th>Question</th>
<th>Scheme</th>
<th>Marks</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a) $z = \frac{150 - 162}{7.5}$</td>
<td>$</td>
<td>z</td>
</tr>
<tr>
<td>$P(F &gt; 150) = P(Z &gt; 1.5) = 0.9452(0071...)$</td>
<td>awrt 0.945</td>
<td>A1 (3)</td>
</tr>
<tr>
<td>(b) $z = \pm \frac{x - 162}{7.5}$</td>
<td>$z = \pm 0.2533$ (or better seen)</td>
<td>B1, M1</td>
</tr>
<tr>
<td>$x = 163.9$</td>
<td>awrt 164</td>
<td>A1 (3)</td>
</tr>
<tr>
<td>(c) $\frac{162 - \mu}{9} = -1.2815515...$</td>
<td>$z = \pm 1.2816$ (or better seen)</td>
<td>B1, M1, A1</td>
</tr>
<tr>
<td>$\mu = 173.533...$</td>
<td>awrt 174</td>
<td>A1 (4)</td>
</tr>
</tbody>
</table>

### Notes

(a) M1 for attempting to standardise with 150, 162 and 7.5. Accept $\pm$

Allow use of symmetry and therefore 174 instead of 150

1st A1 for $\pm 1.6$ seen. Allow 1.6 seen if 174 used or awrt 0.945 is seen. Sight of 0.945(2) is A1.

2nd A1 for awrt 0.945. Do not apply ISW, if 0.9452 is followed by 1 − 0.9452 then award A0

Correct answer only 3/3

(b) B1 for $(z) = \pm 0.2533$ (or better seen).

Giving $z = \pm 0.25$ or $\pm 0.253$ scores B0 here but may get M1/A1

M1 for standardising with $x$ (i.e.), 162 and 7.5, allow $z$ and setting equal to a $z$ value.

Only allow $0.24 \leq z \leq 0.26$. Condense e.g. 160 for 162 etc.

A1 for awrt 164 (Correct answer only scores B0/A1/A1)

(c) B1 for $(z) = \pm 1.2816$ (or better seen).

Allow awrt 1.28 if B0 scored in (b) for $z = \pm 0.25$

M1 for attempting to standardise with 162, 9 and $\mu$, and setting equal to a $z$ value where $1.26 < |z| < 1.31$.

Allow $\pm$ here so signs don’t have to be compatible.

1st A1 for a correct equation with compatible signs and $1.26 < |z| < 1.31$

2nd A1 for awrt 174 (Correct answer only scores B0/A1/A1/A1). Dependent on 1st A1

An equation $\frac{162 - \mu}{9} = 1.2816$ leading to an answer of $\mu - 174$ is A0A0 unless there is clear correct working such as $\frac{162 - 1.2816 \times 9}{9} = ...; \mu = 162 + (162 - x) = 174$ then award A1/A1

NB A common error is: $\frac{162 - \mu}{9} = 1.2816$ followed by $\mu = 162 + 9 \times 1.2816 = awrt 174$. It gets A0A0
### Q2.

<table>
<thead>
<tr>
<th>Question Number</th>
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</tr>
</thead>
<tbody>
<tr>
<td>(a)</td>
<td>$\frac{23 - \mu}{5} = &quot;1.40&quot; \text{ (o.e)}$</td>
<td>awrt ± 1.40 B1 M1A1ft A1</td>
</tr>
<tr>
<td></td>
<td>$\frac{\mu = 16}{16.0}$</td>
<td>(or awrt)</td>
</tr>
<tr>
<td>(b)</td>
<td>0.4192</td>
<td>B1 (4)</td>
</tr>
</tbody>
</table>

#### Notes

(a) B1 for awrt ± 1.40 or better seen anywhere. Condone 1.4 instead of 1.40 M1 for attempting to standardise with 23 and 5 and $\mu$, accept ±

e.g. $\frac{23 - \mu}{25} = 1.40$ can score B1M0 (since using 25 not 5 for standardising)

$\frac{23 - \mu}{5} = 0.9192$ can score B0M1 (since have correct standardisation)

Can accept equivalent equations e.g. $23 - \mu = 5 \times "1.40"$

1st A1ft for standardised expression = to a z value ($|z| > 1$). Signs must be compatible.

Follow through their $z$

e.g. $\frac{23 - \mu}{5} = \text{their } z$ where $z > 1$ or $\frac{\mu - 23}{5} = \text{their } z$ where $z < -1$

2nd A1 for 16 or awrt 16.0 if they are using a more accurate $z$

Correct answer only scores 4/4 but if any working is seen apply scheme

(b) B1 for 0.4192 (but accept 3sf accuracy if 0.9192 - 0.5 is seen)
### Q3.

<table>
<thead>
<tr>
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</tr>
</thead>
<tbody>
<tr>
<td>(a)</td>
<td>(z = \pm \frac{15-16.12}{1.6} = -0.70)</td>
<td>M1 A1</td>
</tr>
<tr>
<td></td>
<td>(P(Z &lt; -0.70) = 1 - 0.7580 = 0.2420)</td>
<td>(awrt 0.242)</td>
</tr>
<tr>
<td>(b)</td>
<td>(P(T &lt; t) = 0.30 \text{ implies} \quad z = \frac{t - 16.12}{1.6} = -0.5244)</td>
<td>M1 A1</td>
</tr>
<tr>
<td></td>
<td>(t = \text{awrt} , 15.28) (allow awrt 15.28/9)</td>
<td></td>
</tr>
</tbody>
</table>

#### Notes

- Allow slips e.g. 16.2 for 16.12 for 1st M1 in (a) and (b)
- 1st M1 for standardising expression with 15, 16.12 and 1.6 - allow ±
- 2nd M1 for 1 - a probability (> 0.5) from tables or calculator based on their standardised value
- Correct answer only scores 3/3

In part (b) they can use any letter or symbol instead of \(t\)
- 1st M1 for standardising with \(t\) (o.e.), 16.12 and 1.6, allow ±, and setting equal to a \(z\) value
- 1st A1 for an equation with \(z = \pm 0.5244\) or better
  - e.g. \(\frac{t - 16.12}{1.6} = \pm 0.52\) (or 0.525) scores M1 (but A0)
- 2nd M1 for solving their linear equation as far as \(t = a \pm b \times 1.6\). Not dependent on 1st M1
  - e.g. solving \(\frac{t - 16.12}{1.6} = 0.3\) to give \(t = 16.12 + 1.6 \times 0.3\) scores this M1
  - Allow \(\frac{t - 16.12}{1.6^2} = 0.3\) to give \(t = 16.12 + 1.6^2 \times 0.3\) to score M1 too
- 2nd A1 dependent on both M marks. Allow awrt 15.28 or awrt 15.29
- Condone awrt 15.3 if a correct expression for \(t = \ldots\) is seen.

**Answers with no working:**
- 15.28 is M1A1M1A1
- 15.29 is M1A0M1A1
- 15.3 is M1A0M1A0

### Q4.

...
<table>
<thead>
<tr>
<th>Question Number</th>
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</tr>
</thead>
</table>
| (a)             | \[ P(X<39) = P \left( Z < \frac{39-30}{5} \right) \]  
|                 | \[ = P(Z < 1.8) = 0.9641 \]  
|                 | (allow awrt 0.964) | M1 A1 (2) |
| (b)             | \[ P(X<d) = P \left( Z < \frac{d-30}{5} \right) = 0.1151 \]  
|                 | \[ 1 - 0.1151 = 0.8849 \]  
|                 | \[ \Rightarrow z = -1.2 \]  
|                 | \[ \Rightarrow \frac{d-30}{5} = -1.2 \]  
|                 | \[ d = 24 \] | M1 B1 M1A1 (4) |
| (c)             | \[ P(X>e) = 0.1151 \] so \[ e = \mu + (\mu - \text{their} \ d) \] or \[ \frac{e-30}{5} = 1.2 \] or \[ \frac{e}{5} = 36 \] | M1 A1 (2) |
| (d)             | \[ P(d < X < e) = 1 - 2 \times 0.1151 \]  
|                 | \[ = 0.7098 \]  
|                 | (AWRT 0.770) | M1 A1 (2) [10] |

Answer only scores all marks in each section BUT check (b) and (c) are in correct order

(a) M1 for standardising with \( \sigma \), \[ z = \frac{39-30}{5} \] is OK  
A1 for 0.9641 or awrt 0.964 but if they go on to calculate \( 1 - 0.9641 \) they get M1A0  

(b) \( 1^\text{st} \) M1 for attempting \( 1 - 0.1151 \). Must be seen in (b) in connection with finding \( d \)  
B1 for \( z = \pm 1.2 \). They must state \( z = \pm 1.2 \) or imply it is a \( z \) value by its use.  
This mark is only available in part (b).  
\( 2^\text{nd} \) M1 for \( \frac{d-30}{5} \) = their negative \( z \) value (or equivalent)  

(c) M1 for a full method to find \( e \). If they used \( z = 1.2 \) in (b) they can get M1 for \( z = \pm 1.2 \) here  
If they use symmetry about the mean \( \mu + (\mu - \text{their} \ d) \) then fit their \( d \) for M1  
Must explicitly see the method used unless the answer is correct.  

(d) M1 for a complete method or use of a correct expression e.g. “their 0.8849” - 0.1151  
or If their \( d < \text{their} \ e \) using their values with \( P(X < e) - P(X < d) \)  
If their \( d > \text{their} \ e \) then they can only score from an argument like \( 1 - 2 \times 0.1151 \)  
A negative probability or probability \( > 1 \) for part (d) scores M0A0.
<table>
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<tbody>
<tr>
<td>(a)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>[ P(X &gt; 168) = P \left( Z &gt; \frac{168 - 160}{5} \right) ]</td>
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<tr>
<td></td>
<td></td>
<td>M1</td>
</tr>
<tr>
<td></td>
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<td>A1</td>
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<tr>
<td></td>
<td></td>
<td>0.0548</td>
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<tr>
<td></td>
<td></td>
<td>awrt 0.0548</td>
</tr>
<tr>
<td>(b)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>[ P(X &lt; w) = P \left( Z &lt; \frac{w - 160}{5} \right) ]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>[ \frac{w - 160}{5} = -2.3263 ]</td>
<td></td>
<td>M1 B1</td>
</tr>
<tr>
<td>[ w = 148.37 ]</td>
<td></td>
<td></td>
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<td></td>
<td></td>
<td>A1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>awrt 148</td>
</tr>
<tr>
<td>(c)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>[ \frac{160 - \mu}{\sigma} = 2.3263 ]</td>
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<td></td>
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<tr>
<td>[ \frac{152 - \mu}{\sigma} = -1.2816 ]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>[ 160 - \mu = 2.3263\sigma ]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>[ 152 - \mu = -1.2816\sigma ]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>[ 8 = 3.6079\sigma ]</td>
<td></td>
<td></td>
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<tr>
<td>[ \sigma = 2.21 \ldots ]</td>
<td></td>
<td></td>
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<tr>
<td>[ \mu = 154.84 \ldots ]</td>
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<tr>
<td></td>
<td></td>
<td>awrt 2.22</td>
</tr>
<tr>
<td></td>
<td></td>
<td>awrt 155</td>
</tr>
</tbody>
</table>

**Notes**

(a)  M1 for an attempt to standardize 168 with 160 and 5 i.e. \( \pm \frac{168 - 160}{5} \) or implied by 1.6

1st A1 for \( P(Z > 1.6) \) or \( P(Z < -1.6) \) i.e. \( z = 1.6 \) and a correct inequality or 1.6 on a shaded diagram

Correct answer to (a) implies all 3 marks

(b)  M1 for attempting \( \pm \left( \frac{w - 160}{5} \right) \) recognizable \( z \) value \( (|z| > 1) \)

B1 for \( z = \pm 2.3263 \) or better. Should be \( z = \ldots \) or implied so: \( 1 - 2.3263 = \frac{w - 160}{5} \) is M0B0

A1 for awrt 148. This may be scored for other \( z \) values so M1B0A1 is possible

For awrt 148 only with no working seen award M1B0A1

(c)  M1 for attempting to standardize 160 or 152 with \( \mu \) and \( \sigma \) (allow \( \pm \)) and equate to \( z \) value \( (|z| > 1) \)

1st B1 for awrt \( \pm 2.33 \) or \( \pm 2.32 \) seen

2nd B1 for awrt \( \pm 1.28 \) seen

2nd M1 for attempt to solve their two linear equations in \( \mu \) and \( \sigma \) leading to equation in just one variable

1st A1 for \( \sigma = \text{awrt 2.22} \). Award when 1st seen

2nd A1 for \( \mu = \text{awrt 155} \). Correct answer only for part (c) can score all 6 marks.

NB \( \sigma = 2.21 \) commonly comes from \( z = 2.34 \) and usually scores M1B0B1M1A0A1

The A marks in (c) require both M marks to have been earned
<table>
<thead>
<tr>
<th>Question Number</th>
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<tbody>
<tr>
<td>(a)</td>
<td>bell shaped, must have inflexions</td>
<td>B1</td>
</tr>
<tr>
<td></td>
<td>154, 172 on axis</td>
<td>B1</td>
</tr>
<tr>
<td></td>
<td>5% and 30%</td>
<td>B1</td>
</tr>
<tr>
<td>(b)</td>
<td>( \mu = 154 + 1.6449\sigma ) <strong>given</strong></td>
<td>B1</td>
</tr>
<tr>
<td></td>
<td>( P(X &lt; 154) = 0.05 )</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>( P \left( \frac{154 - \mu}{\sigma} = 1.6449 \right) )</td>
<td>B1</td>
</tr>
<tr>
<td></td>
<td>( \frac{154 - \mu}{\sigma} = -1.6449 ) or ( \frac{\mu - 154}{\sigma} = 1.6449 )</td>
<td>B1</td>
</tr>
<tr>
<td>(c)</td>
<td>172 - ( \mu = 0.5244\sigma ) or ( \frac{172 - \mu}{\sigma} = 0.5244 ) (allow ( z = 0.52 ) or better here but must be in an equation)</td>
<td>B1</td>
</tr>
<tr>
<td></td>
<td>Solving gives ( \sigma = 8.2976075 ) (awrt 8.30) and ( \mu = 167.64873 ) (awrt 168)</td>
<td>M1 A1 A1</td>
</tr>
<tr>
<td>(d)</td>
<td>( P(\text{Taller than 160cm}) = P\left( \frac{Z &gt; 160 - \mu}{\sigma} \right) )</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>= P(Z &lt; \frac{0.9217994}{\sigma})</td>
<td>B1</td>
</tr>
<tr>
<td></td>
<td>= 0.8212 \text{ awrt 0.82}</td>
<td>A1</td>
</tr>
</tbody>
</table>

Total \( (3) \)

(a) 2nd B1 for 154 and 172 marked but 154 must be \( \mu \) and 172 > \( \mu \). But \( \mu \) need not be marked. Allow for \( \frac{154 - \mu}{\sigma} \) and \( \frac{172 - \mu}{\sigma} \) marked on appropriate sides of the peak.

3rd B1 the 5% and 30% should be clearly indicated in the correct regions i.e. LH tail and RH tail.

(b) M1 for \( \pm \frac{(154 - \mu)}{\sigma} = z \text{ value (z must be recognizable e.g. 1.64, 1.65, 1.96 but NOT 0.5199 etc)} \)

1st B1 for \( \pm 1.6449 \) seen in a line before the final answer.

Also for no incorrect statements (in \( \mu, \sigma \)) equating a \( z \) value and a probability or incorrect signs e.g. \( \frac{154 - \mu}{\sigma} = 0.05 \) or \( \frac{154 - \mu}{\sigma} = 1.6449 \) or \( P(Z < \frac{243}{\sigma}) = 1.6449 \)

(c) B1 for a correct 2nd equation (NB 172 - \( \mu = 0.525\sigma \) is B0, since \( z \) is incorrect)

M1 for solving their two linear equations leading to \( \mu = ... \) or \( \sigma = ... \)

1st A1 for \( \sigma = \text{awrt 8.30, 2nd A1 for } \mu = \text{awrt 168 [NB the 168 can come from false working. These A marks require use of correct equation from (b), and a } z \text{ value for "0.5244" in (c)] \}

NB use of \( z = 0.52 \) will typically get \( \sigma = 8.31 \) and \( \mu = 167.67 \ldots \) and score B1M1A0A1

No working and both correct scores 4/4, only one correct scores 0/4

Provided the M1 is scored the A1s can be scored even with B0 (e.g. for \( z = 0.525 \))

(d) M1 for attempt to standardise with 160, their \( \mu \) and their \( \sigma (> 0) \). Even allow with symbols \( \mu \) and \( \sigma \).

1st B1 for \( \pm 0.92 \)

No working and a correct answer can score 3/3 provided \( \sigma \) and \( \mu \) are correct to 2sf
<table>
<thead>
<tr>
<th>Question Number</th>
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<tbody>
<tr>
<td>(a)</td>
<td>Let the random variable $X$ be the lifetime in hours of bulb. $P(X &lt; 830) = P\left(Z &lt; \frac{830 - 850}{50}\right)$ Standardising with 850 and 50 $= P(Z &lt; -0.4)$ $= 1 - P(Z &lt; 0.4)$ Using 1-(probability $&gt; 0.5$) $= 1 - 0.6554$ $= 0.3446$ or 0.344578 by calculator awrt 0.345</td>
<td>M1</td>
</tr>
<tr>
<td>(b)</td>
<td>$0.3446 \times 500$ $= 172.3$ Accept 172.3 or 172 or 173</td>
<td>M1</td>
</tr>
<tr>
<td>(c)</td>
<td>Standardise with 860 and $\sigma$ and equate to $z$ value $\frac{+818 - 860}{\sigma} = z$ value $\frac{818 - 860}{\sigma} = -0.84(16)$ or $\frac{860 - 818}{\sigma} = 0.84(16)$ or equiv. $\frac{902 - 860}{\sigma} = 0.84(16)$ or equiv. $\sigma = 49.9$ ±0.8416(2) 50 or awrt 49.9</td>
<td>M1</td>
</tr>
<tr>
<td>(d)</td>
<td>Company $Y$ as the mean is greater for $Y$. They have (approximately) the same standard deviation or sd</td>
<td>B1</td>
</tr>
</tbody>
</table>

Notes
- 8(a) If 1-$z$ used e.g. 1-0.4=0.6 then award second M0
- 8(c) M1 can be implied by correct line 2
- A1 for completely correct statement or equivalent.
- Award B1 if 0.8416(2) seen
- Do not award final A1 if any errors in solution e.g. negative sign lost.
- 8(d) Must use statistical terms as underlined.
Q8.

<table>
<thead>
<tr>
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</tr>
</thead>
<tbody>
<tr>
<td>(a) $\frac{127-100}{15}$</td>
<td>M1</td>
<td></td>
</tr>
<tr>
<td>So $P(L &gt; 127) = P(Z &gt; 1.8)$ or $1 - P(Z &lt; 1.8)$ o.e.</td>
<td>A1</td>
<td></td>
</tr>
<tr>
<td>$= 1 - 0.9641 = 0.0359$ (awrt 0.0359)</td>
<td>A1</td>
<td>(3)</td>
</tr>
<tr>
<td>(b) $d = \frac{-100}{15} = -1.2816$ (Calculator gives $-1.2815515...$)</td>
<td>M1, B1</td>
<td></td>
</tr>
<tr>
<td>$d = 80.776$ (awrt 80.8)</td>
<td>A1</td>
<td>(3)</td>
</tr>
<tr>
<td>(c) Require $P(L &gt; 133 \mid L &gt; 127)$</td>
<td>M1</td>
<td></td>
</tr>
<tr>
<td>$= \left[ \frac{P(L &gt; 133)}{P(L &gt; 127)} \right] = \frac{P(Z &gt; 2.2)}{P(L &gt; 127)}$</td>
<td>dM1</td>
<td></td>
</tr>
<tr>
<td>$= \left[ \frac{1 - 0.9861}{1 - 0.9641} \right] = \frac{0.0139}{0.0359}$</td>
<td>A1</td>
<td></td>
</tr>
<tr>
<td>$= 0.3871...$ = awrt 0.39</td>
<td>A1</td>
<td>(4)</td>
</tr>
<tr>
<td>S.C. An attempt at $P(L &lt; 133 \mid L &gt; 127)$ that leads to awrt 0.61 (M0M1A0A0)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Notes

(a) M1 for attempting to standardise with 127, 100 and 15. Allow ±
1st A1 for $Z > 1.8$. Allow a diagram but must have 1.8 and correct area indicated.
Must have the $Z$ so $P(L > 127)$ with or without a diagram is insufficient. May be implied by 0.0359
2nd A1 for awrt 0.0359 (calc. gives 0.035930266...). Correct ans only 3/3. M1A0A1 not poss.

(b) M1 for an attempt to standardise with 100 and 15 and set = ± any $z$ value ($|z| > 1$)
B1 for $z = \pm 1.2816$ (or better) seen anywhere [May be implied by $80.776$ (72...) or better seen]
A1 for awrt 80.8 (can be scored for using 1.28 but then they get M1B0A1)
The 80.8 must follow from correct working.
Calc
If answer is awrt 80.8 and awrt 80.777 or 80.776... or better seen then award M1B1A1
If answer is awrt 80.8 or 80.77 then award M1B0A1 (unless of course $z = 1.2816$ is seen)

(c) 1st M1 for clear indication of correct conditional probability or attempt at correct ratio
So clear attempt at $\frac{P(L > 133)}{P(L > 127)}$ is sufficient for the 1st M1
2nd dM1 dependent on 1st M1 for $P(L > 133)$ leading to $P(Z > 2.2)$
1st A1 for 0.0139 or better seen coming from $P(Z > 2.20)$. Dependent on both Ms
2nd A1 for awrt 0.39. Both Ms required

ALT
If they assume Alice did not check that the phone was working you may see:
$[P(L < 127) \cdot 0] + P(L > 127) \cdot P(L > 133 \mid L > 127)$
Provided the conditional probability is seen as part of this calculation the 1st M1 can be scored and their final answer will be 0.0139(4/4)
An answer of 0.0139 without sight of the conditional probability is 0/4.
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<td>(a) [ \frac{200 - \mu}{7.8} = -1.2816 ] [ \mu = 209.996 \ldots ] [calc gives 1.28155156...] awrt 210</td>
<td>M1 B1</td>
<td>(3)</td>
</tr>
<tr>
<td>[ P(X &gt; 225) = P \left( Z &gt; \frac{225 - &quot;210&quot;}{7.8} \right) ] [ = P(Z &gt; 1.92) \text{ or } 1 - P(Z &lt; 1.92) ] [allow 1.93] [ = 1 - 0.9726 = 0.0274 \text{ (or better)} ] [calc gives 0.0272037...] = awrt 2.7% allow 0.027</td>
<td>M1 A1</td>
<td></td>
</tr>
<tr>
<td>(e) [ \frac{210 - 205}{\sigma} = 2.3263 ] or [ \frac{200 - 205}{\sigma} = -2.3263 ] [calc gives 2.3263478...]</td>
<td>M1 B1</td>
<td>(4)</td>
</tr>
<tr>
<td>[ \sigma = \frac{5}{2.3263} ] [ \sigma = 2.15 (2.14933\ldots) ]</td>
<td>dM1 A1</td>
<td></td>
</tr>
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</table>

Notes
Condone poor handling of notation if answers are correct but A marks must have correct working.

(a) M1 for an attempt to standardise (allow \( \pm \)) with 200 and 7.8 and set \( = \pm \) any \( z \) value (\( |z| > 1 \))
B1 for \( z = \pm 1.2816 \) (or better used as \( z \))[May be implied by 209.996(102\ldots) or better seen]
A1 for awrt 210 (can be scored for using 1.28 but then they get M1B0A1)
The 210 must follow from correct working—sign scores A0
If answer is awrt 210 and 209.996... or better seen then award M1B1A1
\( z = 1.28 \) gives 209.984 and \( z = 1.282 \) gives 209.9906 and both score M1B0A1
If answer is awrt 210 or awrt 209.996 then award M1B0A1 (unless of course \( z = 1.2816 \) is seen)

(b) M1 for attempting to standardise with 225, their mean and 7.8. Allow \( \pm \)
1st A1 for \( Z > \text{awrt} 1.92/3. \) Allow a diagram but must have 1.92/3 and correct area indicated.
Must have the \( Z \) so \( P(X > 225) \) with or without a diagram is not sufficient.
Award for 1 - 0.9726 or 1 - 0.9732
2nd A1 for 2.7% or better (calculator gives 2.72...) Allow awrt 0.027. Correct ans scores 3/3

(c) 1st M1 for an attempt to standardise with 200 or 210, 205 and \( \sigma \) and set \( = \pm \) any \( z \) value (\( |z| > 2 \))
B1 for \( z = 2.3263 \) (or better) and compatible signs.
If B0 in (a) for using a value in \( [1.28, 1.29] \) but not using 1.2816: allow awrt 2.33 here
2nd dM1 Dependent on the first M1 for correctly rearranging to make \( \sigma = \ldots \) May be implied e.g. \( \frac{1}{2} = 2.32 \rightarrow \sigma = 2.16 \) (M1A0) BUT must have \( \sigma > 0 \)
A1 for awrt 2.15. Must follow from correct working but a range of possible \( z \) values will do.
NB. 2.320 < \( z < 2.331 \) will give an answer of awrt 2.15
Q10.

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| (a)             | The random variable $H \sim$ height of females  
$P(H > 170) = P\left(Z > \frac{170 - 160}{8}\right) \quad [= P(Z > 1.25)]$  
$= 1 - 0.8944$  
$= 0.1056 \quad \text{ (calc} \ 0.1056498\ldots\text{)} \quad \text{awrt} \ 0.106 \ (\text{accept} \ 10.6\%)$ | M1 M1 A1 |
| (b)             | $P(H > 180) = P\left(Z > \frac{180 - 160}{8}\right) \quad [=1 - 0.9938]$  
$= 0.0062 \quad \text{ (calc} \ 0.006209\ldots\text{)} \quad \text{awrt} \ 0.0062 \text{ or } \frac{0.0}{0.000}$  
$[P(H > 180| H > 170)] = \frac{0.0062}{0.1056}$  
$= 0.0587 \quad \text{ (calc} \ 0.0587760\ldots\text{)} \quad \text{awrt} \ 0.0587 \text{ or } 0.0588$ | M1 A1 M1 |
| (c)             | $P(H > h | H > 170) \ (0.5)$  
$[P(H > h)] = 0.5 \times 0.1056 = 0.0528 \quad \text{(calc} \ 0.0528249\ldots\text{)} \quad \text{or} \ [P(H < h)] = 0.9472$  
$rac{h - 160}{8} = 1.62 \quad \text{(calc} \ 1.6180592\ldots\text{)}$  
$h = \text{awrt} \ 173 \text{ cm}$ | M1 A1 B1 |

Notes

(a) 1st M1 for attempt at standardising with 170, 160 and 8. Allow ± i.e. for $\frac{170 - 160}{8}$  
2nd M1 for attempting 1 − $p$ where 0.8 < $p$ < 1. Correct answer only 3/3
(b) 1st M1 for standardising with 180, 160 and 8  
1st A1 for 0.0062 seen, maybe seen as part of another expression/calculation.  
2nd M1 using conditional probability with denom = their (a) and num < their denom Values needed.  
2nd A1 for awrt 0.0587 or 0.0588. Condone 5.87% or 5.88% or $\frac{1}{10}$.  
Correct answer only 4/4
(c) 1st M1 for a correct conditional probability statement. Either line and don’t insist on 0.5, ft (a)  
1st A1ft for $[P(H > h)] = 0.5 \times \text{their}(a)$  
Award M1A1ft for correct evaluation of 0.5×their(a) or sight of 0.0528 or better  
2nd M1 for attempt to standardise (+) with 160 and 8 and set equal to ± $z$ value (1.56 < $|z|$ < 1.68)  
B1 for ($z$ =) awrt $\pm 1.62$ (seen)  
2nd A1 for awrt 173 but dependent on both M marks.