Write your name here

Surname	Other names

Malvern College
U6th Trials

Centre Number

Candidate Number

February 2020

Advanced
Further Mathematics Option 1
Paper 3: Further Pure Mathematics 1

Time: 1 hour 30 minutes

Paper Reference
9FM0/3A

You must have:
Mathematical Formulae and Statistical Tables, calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for algebraic manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 8 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over ▶

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Pearson

Answer ALL questions. Write your answers in the spaces provided.

1. Use Simpson's Rule with 6 intervals to estimate

$$\int_1^4 \sqrt{1 + x^3}\,\mathrm{d}x$$

(5)

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Question 1 continued

(Total for Question 1 is 5 marks)

2. Given k is a constant and that

$$y = x^3 e^{kx}$$

use Leibnitz theorem to show that

$$\frac{d^n y}{dx^n} = k^{n-3}e^{kx}\left(k^3x^3 + 3nk^2x^2 + 3n(n-1)kx + n(n-1)(n-2)\right)$$

(4)

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Question 2 continued

(Total for Question 2 is 4 marks)

3. A vibrating spring, fixed at one end, has an external force acting on it such that the centre of the spring moves in a straight line. At time t seconds, $t \geqslant 0$, the displacement of the centre C of the spring from a fixed point O is x micrometres.

The displacement of C from O is modelled by the differential equation

$$t^2 \frac{d^2x}{dt^2} - 2t\frac{dx}{dt} + (2 + t^2)x = t^4 \qquad \text{(I)}$$

(a) Show that the transformation $x = tv$ transforms equation (I) into the equation

$$\frac{d^2v}{dt^2} + v = t \qquad \text{(II)}$$

(5)

(b) Hence find the general equation for the displacement of C from O at time t seconds.

(7)

(c) (i) State what happens to the displacement of C from O as t becomes large.

(ii) Comment on the model with reference to this long term behaviour.

(2)

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Question 3 continued

(Total for Question 3 is 14 marks)

4.

$$\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + y = 0 \qquad \text{(I)}$$

(a) Show that

$$\frac{d^5y}{dx^5} = ax\frac{d^4y}{dx^4} + b\frac{d^3y}{dx^3}$$

where a and b are integers to be found.

(4)

(b) Hence find a series solution, in ascending powers of x, as far as the term in x^5, of the differential equation (I) where $y = 0$ and $\frac{dy}{dx} = 1$ at $x = 0$

(5)

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Question 4 continued

(Total for Question 4 is 9 marks)

5. The normal to the parabola $y^2 = 4ax$ at the point $P(ap^2, 2ap)$ passes through the parabola again at the point $Q(aq^2, 2aq)$.

The line OP is perpendicular to the line OQ, where O is the origin.

Prove that $p^2 = 2$

(9)

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Question 5 continued

(Total for Question 5 is 9 marks)

6. A tetrahedron has vertices $A(1, 2, 1)$, $B(0, 1, 0)$, $C(2, 1, 3)$ and $D(10, 5, 5)$.

Find

(a) a Cartesian equation of the plane ABC.

(3)

(b) the volume of the tetrahedron $ABCD$.

(3)

The plane Π has equation $2x - 3y + 3 = 0$

The point E lies on the line AC and the point F lies on the line AD.

Given that Π contains the point B, the point E and the point F,

(c) find the value of k such that $\overrightarrow{AE} = k\overrightarrow{AC}$.

(3)

Given that $\overrightarrow{AF} = \frac{1}{9}\overrightarrow{AD}$

(d) show that the volume of the tetrahedron $ABCD$ is 45 times the volume of the tetrahedron $ABEF$.

(2)

Question 6 continued

(Total for Question 6 is 11 marks)

7. P and Q are two distinct points on the ellipse described by the equation $x^2 + 4y^2 = 4$

The line l passes through the point P and the point Q.

The tangent to the ellipse at P and the tangent to the ellipse at Q intersect at the point (r, s).

Show that an equation of the line l is

$$4sy + rx = 4$$

(8)

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Question 7 continued

(Total for Question 7 is 8 marks)

8.

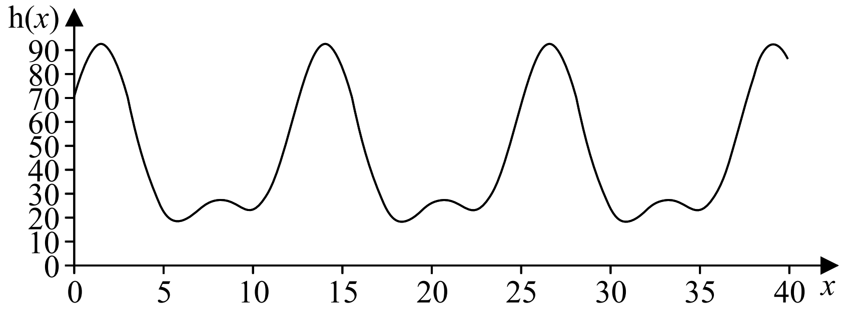


Figure 1

Figure 1 shows the graph of the function h(x) with equation

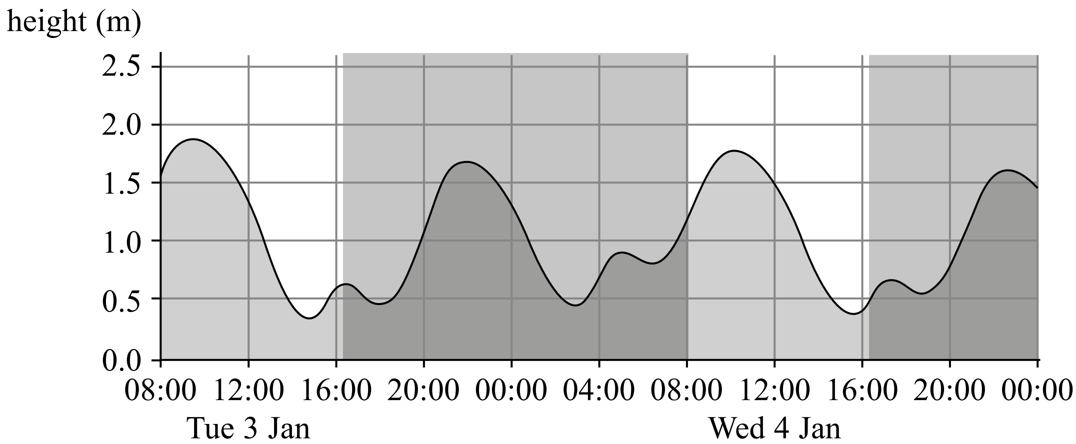
$$\mathrm{h}(x) = 45 + 15\sin x + 21\sin\left(\frac{x}{2}\right) + 25\cos\left(\frac{x}{2}\right) \qquad x \in [0, 40]$$

(a) Show that

$$\frac{\mathrm{d}h}{\mathrm{d}x} = \frac{(t^2 - 6t - 17)(9t^2 + 4t - 3)}{2(1 + t^2)^2}$$

where $t = \tan\left(\frac{x}{4}\right)$.

(6)



Source: [1]Data taken on 29th December 2016 from http://www.ukho.gov.uk/easytide/EasyTide

Figure 2

Figure 2 shows a graph of predicted tide heights, in metres, for Portland harbour from 08:00 on the 3rd January 2017 to the end of the 4th January 2017[1].

The graph of kh(x), where k is a constant and x is the number of hours after 08:00 on 3rd of January, can be used to model the predicted tide heights, in metres, for this period of time.

(b) (i) Suggest a value of k that could be used for the graph of kh(x) to form a suitable model.

(ii) Why may such a model be suitable to predict the times when the tide heights are at their peaks, but not to predict the heights of these peaks?

(3)

(c) Use Figure 2 and the result of part (a) to estimate, to the nearest minute, the time of the highest tide height on the 4th January 2017.

(6)

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Question 8 continued

Question 8 continued

(Total for Question 8 is 15 marks)

TOTAL FOR PAPER IS 75 MARKS