1 Find $\frac{d\theta}{dt}$ where $\theta = t^2 - 3t$.

2 Find $\frac{dA}{dr}$ where $A = 2\pi r$.

3 Given that $r = \frac{12}{t}$, find the value of $\frac{dr}{dt}$ when $t = 3$.

4 The surface area, A cm², of an expanding sphere of radius r cm is given by $A = 4\pi r^2$. Find the rate of change of the area with respect to the radius at the instant when the radius is 6 cm.

5 The displacement, s metres, of a car from a fixed point at time t seconds is given by $s = t^2 + 8t$. Find the rate of change of the displacement with respect to time at the instant when $t = 5$.

(P) **6** A rectangular garden is fenced on three sides, and the house forms the fourth side of the rectangle.

a Given that the total length of the fence is 80 m, show that the area, A, of the garden is given by the formula $A = y(80 - 2y)$, where y is the distance from the house to the end of the garden.

b Given that the area is a maximum for this length of fence, find the dimensions of the enclosed garden, and the area which is enclosed.

(P) **7** A closed cylinder has total surface area equal to 600π.

a Show that the volume, V cm³, of this cylinder is given by the formula $V = 300\pi r - \pi r^3$, where r cm is the radius of the cylinder.

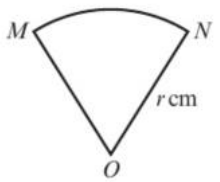
b Find the maximum volume of such a cylinder.

(P) **8** A sector of a circle has area 100 cm².

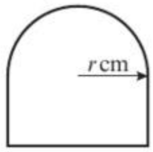
a Show that the perimeter of this sector is given by the formula

$$P = 2r + \frac{200}{r},\ r > \sqrt{\frac{100}{\pi}}$$

b Find the minimum value for the perimeter.



(E/P) **9** A shape consists of a rectangular base with a semicircular top, as shown.



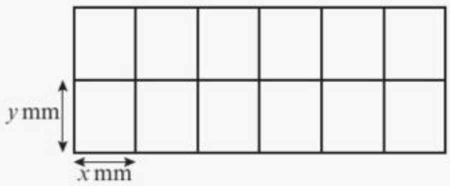
a Given that the perimeter of the shape is 40 cm, show that its area, A cm², is given by the formula

$$A = 40r - 2r^2 - \frac{\pi r^2}{2}$$

where r cm is the radius of the semicircle. **(2 marks)**

b Hence find the maximum value for the area of the shape. **(4 marks)**

(E/P) **10** The shape shown is a wire frame in the form of a large rectangle split by parallel lengths of wire into 12 smaller equal-sized rectangles.



a Given that the total length of wire used to complete the whole frame is 1512 mm, show that the area of the whole shape, A mm², is given by the formula

$$A = 1296x - \frac{108x^2}{7}$$

where x mm is the width of one of the smaller rectangles. **(4 marks)**

b Hence find the maximum area which can be enclosed in this way. **(4 marks)**

Differentiation 12K

1 $\theta = t^2 - 3t$

$\frac{d\theta}{dt} = 2t - 3$

2 $A = 2\pi r$

$\frac{dA}{dr} = 2\pi$

3 $r = \frac{12}{t} = 12t^{-1}$

$\frac{dr}{dt} = -12t^{-2} = -\frac{12}{t^2}$

When $t = 3$,

$\frac{dr}{dt} = -\frac{12}{3^2} = -\frac{12}{9} = -\frac{4}{3}$

4 $A = 4\pi r^2$

$\frac{dA}{dr} = 8\pi r$

When $r = 6$,

$\frac{dA}{dr} = 8\pi \times 6$

$= 48\pi$

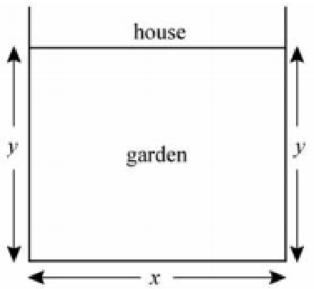
5 $s = t^2 + 8t$

$\frac{ds}{dt} = 2t + 8$

When $t = 5$,

$\frac{ds}{dt} = 2 \times 5 + 8 = 18$

6 a



Let the width of the garden be x m.

Then $x + 2y = 80$

$x = 80 - 2y$ (1)

Area $A = xy$

$= y(80 - 2y)$

$= 80y - 2y^2$

6 b $\frac{dA}{dy} = 80 - 4y$

Putting $\frac{dA}{dy} = 0$ for maximum area:

$80 - 4y = 0$

$y = 20$

Substituting in (1): $x = 40$

So area = 40 m × 20 m = 800 m^2

7 a Total surface area = $2\pi rh + 2\pi r^2$

$2\pi rh + 2\pi r^2 = 600\pi$

$rh = 300 - r^2$

Volume = $\pi r^2 h = \pi r(rh) = \pi r(300 - r^2)$

So $V = 300\pi r - \pi r^3$

b For maximum volume, $\frac{dV}{dr} = 0$

$\frac{dV}{dr} = 300\pi - 3\pi r^2$

$300\pi - 3\pi r^2 = 0$

$r^2 = 100$

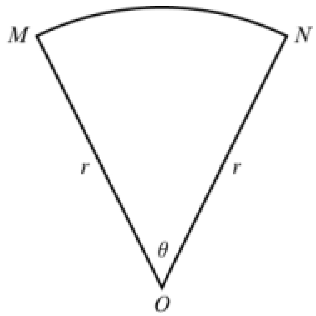
$r = 10$

Substituting $r = 10$ into V gives:

$V = 300\pi \times 10 - \pi \times 10^3 = 2000\pi$

Maximum volume = 2000π cm^3

8 a



Let angle $MON = \theta$ radians

Then perimeter $P = 2r + r\theta$ **(1)**

and area $A = \frac{1}{2}r^2\theta$

Area = 100 cm^2

$\frac{1}{2}r^2\theta = 100$

$r\theta = \frac{200}{r}$

Substituting into **(1)** gives:

$P = 2r + \frac{200}{r}$ **(2)**

8 a Since area of circle > area of sector

$$\pi r^2 > 100$$

$$r > \sqrt{\frac{100}{\pi}}$$

b For minimum perimeter, $\frac{dP}{dr} = 0$

$$\frac{dP}{dr} = 2 - \frac{200}{r^2}$$

$$2 - \frac{200}{r^2} = 0$$

$$r^2 = 100$$

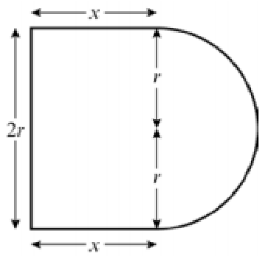
$$r = 10$$

Substituting into (2) gives:

$$P = 20 + \frac{200}{10} = 40$$

Minimum perimeter = 40 cm

9 a



Let the rectangle have dimensions $2r$ by x cm.

Perimeter of figure = $(2r + 2x + \pi r)$ cm

Perimeter = 40 cm, so

$$2r + 2x + \pi r = 40$$

$$x = \frac{40 - \pi r - 2r}{2}$$

Area = rectangle + semicircle

$$= 2rx + \frac{1}{2}\pi r^2$$

Substituting $x = \frac{40 - \pi r - 2r}{2}$:

$$A = r(40 - \pi r - 2r) + \frac{1}{2}\pi r^2$$

$$= 40r - 2r^2 - \frac{1}{2}\pi r^2$$

b For maximum area, $\frac{dA}{dr} = 0$:

$$\frac{dA}{dr} = 40 - 4r - \pi r$$

$$40 - 4r - \pi r = 0$$

9 b $r = \frac{40}{4 + \pi}$

When $r = \frac{40}{4+\pi}$,

$$A = 40 \times \frac{40}{4+\pi} - 2\left(\frac{40}{4+\pi}\right)^2 - \frac{1}{2}\pi\left(\frac{40}{4+\pi}\right)^2$$

$$= \frac{1600}{4+\pi} - \left(2 + \frac{1}{2}\pi\right)\left(\frac{40}{4+\pi}\right)^2$$

$$= \frac{1600}{4+\pi} - \frac{4+\pi}{2} \times \frac{1600}{(4+\pi)^2}$$

$$= \frac{1600}{4+\pi} - \frac{800}{4+\pi}$$

$$= \frac{800}{4+\pi}$$

So maximum area = $\frac{800}{4+\pi}$ cm^2

10 a Total length of wire = $(18x + 14y)$ mm

Length = 1512 mm, so

$$18x + 14y = 1512$$

$$y = \frac{1512 - 18x}{14}$$

Total area A mm^2 is given by:

$$A = 2y \times 6x$$

Substituting $y = \frac{1512 - 18x}{14}$:

$$A = 12x\left(\frac{1512 - 18x}{14}\right)$$

$$= 1296x - \frac{108}{7}x^2$$

b For maximum area $\frac{dA}{dx} = 0$:

$$\frac{dA}{dx} = 1296 - \frac{216}{7}x$$

$$1296 - \frac{216}{7}x = 0$$

$$x = \frac{7 \times 1296}{216} = 42$$

When $x = 42$,

$$A = 1296 \times 42 - \frac{108}{7} \times 42^2$$

$$= 27\,216$$

Maximum area = 27 216 mm^2

(Check: $\frac{d^2A}{dx^2} = -\frac{216}{7} < 0$ $\therefore$ maximum)