L6 HL Kinematics

2 A particle is moving in a straight line. At time t seconds, its displacement, x m, from a fixed point O on the line is given by $x = 2t^3 - 8t$. Find:

a the velocity of the particle when $t = 3$ **b** the acceleration of the particle when $t = 2$.

3 A particle P is moving on the x-axis. At time t seconds (where $t \geqslant 0$) , the velocity of P is v m s^{-1} in the direction of x increasing, where $v = 12 - t - t^2$.

Find the acceleration of P when P is instantaneously at rest.

4 A particle is moving in a straight line. At time t seconds, its displacement, x m, from a fixed point O on the line is given by $x = 4t^3 - 39t^2 + 120t$.

Find the distance between the two points where P is instantaneously at rest.

5 A particle P moves in a straight line. At time t seconds the acceleration of P is a m s^{-2} and the velocity v m s^{-1} is given by $v = kt - 3t^2$, where k is a constant.

The initial acceleration of P is 4 m s^{-2}.

a Find the value of k. **(3 marks)**

b Using the value of k found in part **a**, find the acceleration when P is instantaneously at rest. **(3 marks)**

6 The print head on a printer moves such that its displacement s cm from the side of the printer at time t seconds is given by:

$$\tfrac{1}{4}(4t^3 - 15t^2 + 12t + 30),\ 0 \leqslant t \leqslant 3$$

Find the distance between the points when the print head is instantaneously at rest, in cm to 1 decimal place. **(6 marks)**

2 a $x = 2t^3 - 8t$

$$v = \frac{dx}{dt} = 6t^2 - 8$$

When $t = 3$, $v = 6 \times 3^2 - 8 = 46$

The velocity of the particle when $t = 3$ is 46 m s^{-1}.

b $$a = \frac{dv}{dt} = 12t$$

When $t = 2$, $a = 12 \times 2 = 24$

The acceleration of the particle when $t = 2$ is 24 m s^{-2}.

3 P is at rest when $v = 0$.

$12 - t - t^2 = 0$

$(4 + t)(3 - t) = 0$

$t = -4$ or $t = 3$

$t \geq 0$, so $t = 3$

$$a = \frac{\mathrm{d}v}{\mathrm{d}t} = -1 - 2t$$

When $t = 3$, $a = -1 - 2 \times 3 = -7$

The acceleration of P when P is instantaneously at rest is -7 m s^{-2}, or 7 m s^{-2} in the direction of x decreasing.

4 $x = 4t^3 - 39t^2 + 120t$

$$v = \frac{\mathrm{d}x}{\mathrm{d}t} = 12t^2 - 78t + 120$$

P is at rest when $v = 0$.

$12t^2 - 78t + 120 = 0$

$2t^2 - 13t + 20 = 0$

$(2t - 5)(t - 4) = 0$

P is at rest when $t = 2.5$ and $t = 4$.

When $t = 2.5$, $x = 4(2.5)^3 - 39(2.5)^2 + 120(2.5) = 118.75$

When $t = 4$, $x = 4(4)^3 - 39(4)^2 + 120(4) = 112$

The distance between the two points where P is instantaneously at rest is $118.75 - 112 = 6.75$ m.

5 $v = kt - 3t^2$

a $$a = \frac{\mathrm{d}v}{\mathrm{d}t} = k - 6t$$

When $t = 0$, $a = 4$

$k - 6 \times 0 = 4$

$k = 4$

b P is at rest when $v = 0$.

$4t - 3t^2 = 0$

$t(4 - 3t) = 0$

P is at rest when $t = 0$ and $t = \frac{4}{3}$.

When $t = \frac{4}{3}$, $a = 4 - 6 \times \frac{4}{3} = 4 - 8 = -4$

When P is next at rest, the acceleration is -4 m s^{-2}.

6 $s = \frac{1}{4}(4t^3 - 15t^2 + 12t + 30)$

$v = \dfrac{ds}{dt} = \frac{1}{4}(12t^2 - 30t + 12)$

6 The print head is at rest when $v = 0$.

$\frac{1}{4}(12t^2 - 30t + 12) = 0$

$12t^2 - 30t + 12 = 0$

$2t^2 - 5t + 2 = 0$

$(2t - 1)(t - 2) = 0$

The print head is at rest when $t = 0.5$ and $t = 2$.

When $t = 0.5$,

$s = \frac{1}{4}(4(0.5)^3 - 15(0.5)^2 + 12(0.5) + 30)$

$= \frac{1}{4}(0.5 - 3.75 + 6 + 30)$

$= 8.1875$

When $t = 2$,

$s = \frac{1}{4}(4(2)^3 - 15(2)^2 + 12(2) + 30)$

$= \frac{1}{4}(32 - 60 + 24 + 30)$

$= 6.5$

Distance between these two points $= 8.1875 - 6.5$

$= 1.6875$ cm

$= 1.7$ cm (1 d.p.)

The distance between the points when the print head is instantaneously at rest is 1.7 cm.