

## L6 HL Rates of Change – Change of Variable

1 Given that  $A = \frac{1}{4}\pi r^2$  and that  $\frac{dr}{dt} = 6$ , find  $\frac{dA}{dt}$  when  $r = 2$ .

2 Given that  $y = xe^x$  and that  $\frac{dx}{dt} = 5$ , find  $\frac{dy}{dt}$  when  $x = 2$ .

3 Given that  $r = 1 + 3 \cos \theta$  and that  $\frac{d\theta}{dt} = 3$ , find  $\frac{dr}{dt}$  when  $\theta = \frac{\pi}{6}$

4 Given that  $V = \frac{1}{3}\pi r^3$  and that  $\frac{dV}{dt} = 8$ , find  $\frac{dr}{dt}$  when  $r = 3$ .

1  $A = \frac{1}{4}\pi r^2$   
 $\frac{dA}{dr} = \frac{1}{2}\pi r$   
 $\frac{dA}{dt} = \frac{dA}{dr} \times \frac{dr}{dt}$   
 $= \frac{1}{2}\pi r \times 6 = 3\pi r$   
When  $r = 2$ ,  
 $\frac{dA}{dt} = 3\pi \times 2 = 6\pi$

2  $y = xe^x$   
 $\frac{dy}{dx} = xe^x + e^x = (x+1)e^x$   
 $\frac{dy}{dt} = \frac{dy}{dx} \times \frac{dx}{dt}$   
 $= (x+1)e^x \times 5 = 5(x+1)e^x$   
When  $x = 2$ ,  
 $\frac{dy}{dt} = 5(2+1)e^2 = 15e^2$

3  $r = 1 + 3\cos \theta$   
 $\frac{dr}{d\theta} = -3\sin \theta$   
 $\frac{dr}{dt} = \frac{dr}{d\theta} \times \frac{d\theta}{dt}$   
 $= -3\sin \theta \times 3 = -9\sin \theta$   
When  $\theta = \frac{\pi}{6}$ ,  
 $\frac{dr}{dt} = -9\sin \frac{\pi}{6} = -\frac{9}{2}$

4  $V = \frac{1}{3}\pi r^3$   
 $\frac{dV}{dr} = \pi r^2 \Rightarrow \frac{dr}{dV} = \frac{1}{\pi r^2}$   
 $\frac{dr}{dt} = \frac{dr}{dV} \times \frac{dV}{dt}$   
 $= \frac{1}{\pi r^2} \times 8 = \frac{8}{\pi r^2}$   
When  $r = 3$ ,  
 $\frac{dr}{dt} = \frac{8}{\pi \times 3^2} = \frac{8}{9\pi}$