Proof Questions 2 (IGCSE)

EXERCISE 23.6

1 Prove that the sum of two consecutive integers is always odd.

2 Prove that the product of any two even numbers is always even.

3 Prove that the product of any two odd numbers is always odd.

4 Prove that the sum of three consecutive integers is always a multiple of three.

5 Prove that the difference between the squares of any two odd integers is always divisible by four.

6 The diagram shows a square measuring $(a + b)$ along each side. A smaller square, of side c, is inscribed inside the larger square.

a) Show that the total area of the four triangles is $2ab$.

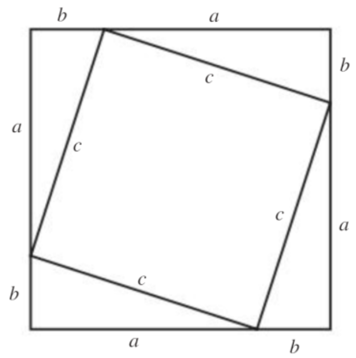
b) Obtain expressions for the total area of the shape in two ways:

(i) by adding together the areas of the four triangles and the inner square

(ii) by expanding $(a + b)^2$.

c) Use your results from part **b)** to prove that $c^2 = a^2 + b^2$.

d) What well-known theorem have you just proved?



7 **a)** Show that $(100x + 1)(100x - 1) = 10\,000x^2 - 1$.

b) Hence show that 89 999 is not prime.

8 By writing the nth term of the sequence 1, 3, 5, 7, ... as $(2n - 1)$, or otherwise, show that the difference between the squares of any two consecutive odd numbers is a multiple of 8. **[Edexcel]**

Exercise 23.6 (page 459)

1 Let the numbers be n and $n + 1$.
Their sum is $2n + 1$ which is odd.

2 Let the numbers be $2n$ and $2m$.
Their product is $4mn = 2 \times 2mn$, hence even.

3 Let the numbers be $2n + 1$ and $2m + 1$.
Their product is $(2n + 1)(2m + 1) = 4mn + 2n + 2m + 1 = 2 \times (2mn + n + m) + 1$, hence odd.

4 Let the numbers be n , $n + 1$ and $n + 2$.
Their sum is $n + n + 1 + n + 2 = 3n + 3$
$= 3 \times (n + 1)$,
hence a multiple of 3.

5 Let the numbers be $2n + 1$ and $2m + 1$.
Then

$$
\begin{aligned}
(2n + 1)^2 - (2m + 1)^2 &= [4n^2 + 4n + 1] \\
&\quad - [4m^2 + 4m + 1] \\
&= 4n^2 + 4n + 1 - 4m^2 - 4m - 1 \\
&= 4n^2 + 4n - 4m^2 - 4m \\
&= 4(n^2 + n - m^2 - m),
\end{aligned}
$$

hence a multiple of 4.

6 a) $4 \times \frac{1}{2}ab = 2ab$
b) i) $c^2 + 2ab$ **ii)** $a^2 + b^2 + 2ab$
d) Pythagoras' theorem

7 b) Setting $x = 3$ gives $301 \times 299 = 89\,999$, so not prime.

8 Let the consecutive odd numbers be $2n - 1$ and $2n + 1$.
Then

$$
\begin{aligned}
(2n + 1)^2 - (2n - 1)^2 &= [4n^2 + 4n + 1] - [4n^2 - 4n + 1] \\
&= 4n^2 + 4n + 1 - 4n^2 + 4n - 1 \\
&= 8n,
\end{aligned}
$$

hence a multiple of 8.