

Tangents and Normals

Exercise 8C

- ① The sketch shows the graph of $y = 5x - x^2$. The marked point, P, has coordinates (3, 6). Work out

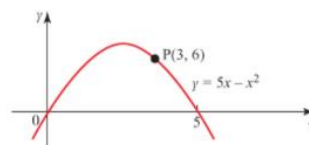


Figure 8.11

- (i) the gradient function $\frac{dy}{dx}$
 - (ii) the gradient of the curve at P
 - (iii) the equation of the tangent at P
 - (iv) the equation of the normal at P
- ② The sketch shows the graph of $y = 3x^2 - x^3$.

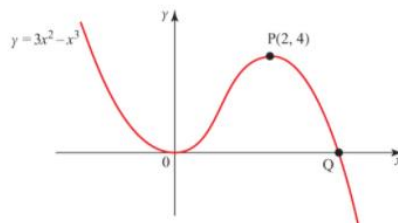


Figure 8.12

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- (i) The marked point, P, has coordinates (2, 4). Work out
 - (a) the equation of the tangent at P
 - (b) the equation of the normal at P
 - (ii) The graph touches the x -axis at the origin O and crosses it at the point Q. Work out the equation of the tangent at Q.
 - (iii) Without further calculation, state the equation of the tangent to the curve at O.
- ③ The sketch shows the graph of $y = x^5 - x^3$.

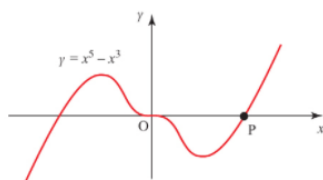


Figure 8.13

- (i) Work out the coordinates of the point P where the curve crosses the positive x -axis.
 - (ii) Work out the equation of the tangent at P.
 - (iii) Work out the equation of the normal at P. The tangent at P meets the y -axis at Q and the normal meets the y -axis at R.
 - (iv) Work out the coordinates of Q and R and hence calculate the area of triangle PQR.
- ④ (i) Given that $y = x^3 - 3x^2 + 4x + 1$, work out the gradient function $\frac{dy}{dx}$.
- (ii) The point P is on the curve $y = x^3 - 3x^2 + 4x + 1$ and its x -coordinate is 2.
 - (a) Work out the equation of the tangent at P
 - (b) Work out the equation of the normal at P
 - (iii) Work out the values of x for which the curve has a gradient of 13

- ⑤ The sketch shows the graph of $y = x^3 - 9x^2 + 23x - 15$

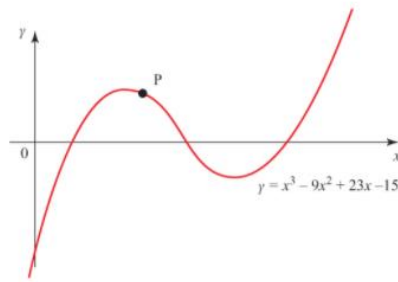


Figure 8.14

The point P marked on the curve has its x -coordinate equal to 2

- (i) Work out the equation of the tangent at P.
 - Q is a point on the curve where the tangent is parallel to the tangent at P.
 - (ii) Work out the equation of the tangent at Q.
- ⑥ The point $(2, -8)$ is on the curve $y = x^3 - px + q$.

- (i) Identify a relationship between p and q .

The tangent to this curve at the point $(2, -8)$ is parallel to the x -axis.

- (ii) Work out the value of p .
 - (iii) Work out the coordinates of the other point where the tangent is parallel to the x -axis.
 - (iv) Work out the equation of the normal to the curve at the point where it crosses the y -axis.
- ⑦ The sketch shows the graph of $y = x^2 - x - 1$

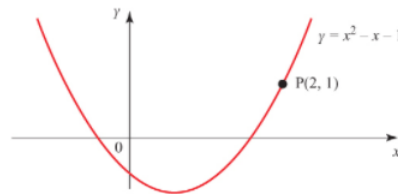


Figure 8.15

- (i) Work out the equation of the tangent at the point P.

The normal at a point Q on the curve is parallel to the tangent at P.

- (ii) Work out the coordinates of the point Q.
- ⑧ A curve has the equation $y = (x - 3)(7 - x)$.
- (i) Work out the equation of the tangent at the point $(6, 3)$.
 - (ii) Work out the equation of the normal at the point $(6, 3)$.
 - (iii) Which one of these lines passes through the origin?

- ⑨ A curve has the equation $y = 1.5x^3 - 3.5x^2 + 2x$.

- (i) Show that the curve passes through the points $(0, 0)$ and $(1, 0)$.
- (ii) Work out the equations of the tangents and normals at each of these points.
- (iii) What shape is formed by the four lines in part (ii)?

- ⑩ Figure 8.16 shows the curve with the equation $y = x^2 + \frac{2}{x}$ for $x > 0$

- (i) Work out the gradient function $\frac{dy}{dx}$ and calculate the coordinates of the minimum point.
- (ii) State the equations of the tangent and the normal at that minimum point.
- (iii) Work out the equations of the tangent and normal at the point where $x = 2$

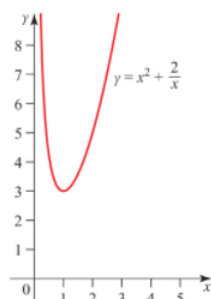


Figure 8.16