

Graph Sketching from Gradients

① For each of the curves given below

(a) work out $\frac{dy}{dx}$ and the value(s) of x for which $\frac{dy}{dx} = 0$

(b) work out the value(s) of $\frac{d^2y}{dx^2}$ at those points

(c) classify the point(s) on the curve with these x -values

(d) work out the corresponding y -value(s)

(e) sketch the curve.

(i) $y = 1 + x - 2x^2$

(iii) $y = 12x + 3x^2 - 2x^3$

(iii) $y = x^3 - 4x^2 + 9$

(iv) $y = x(x - 1)^2$

(v) $y = x^2(x - 1)^2$

(vi) $y = x^3 - 48x$

(vii) $y = x^3 + 6x^2 - 36x + 25$

(viii) $y = 2x^3 - 15x^2 + 24x + 8$

Activity 8.6 (page 226)

When $x = 0^\circ$ the gradient is zero. It then decreases through negative values reaching its most negative value when $x = 90^\circ$. It increases to zero when $x = 180^\circ$ and continues to increase through positive values until it is greatest when $x = 270^\circ$. The gradient then decreases to zero when $x = 360^\circ$.

Discussion point (page 228)

There are no more values when $\frac{dy}{dx} = 0$, so there are no more turning points. As x increases

beyond the point where $x = 2$, $\frac{dy}{dx}$ takes positive values and so the curve will cross the x -axis again. To the left of $x = -2$ the gradient is always negative, giving a further point of intersection with the x -axis.

Discussion point (page 228)

- (i) The curve crosses the x -axis when $x^3 - 12x + 3 = 0$. This does not factorise, so the values of x cannot be found easily.
- (ii) Only when the equation obtained when $y = 0$ factorises.

Exercise 8F (page 234)

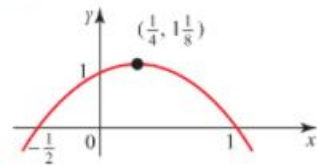
1 (i) (a) $\frac{dy}{dx} = 1 - 4x$; $x = \frac{1}{4}$

(b) $\frac{d^2y}{dx^2} = -4$

(c) Max

(d) $y = 1\frac{1}{8}$

(e)



(ii) (a) $\frac{dy}{dx} = 12 + 6x - 6x^2$;
 $x = -1, x = 2$

(b) $\frac{d^2y}{dx^2} = 6 - 12x$.

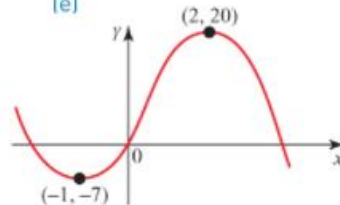
When $x = -1, \frac{d^2y}{dx^2} = 18$

When $x = 2, \frac{d^2y}{dx^2} = -18$

(c) Min when $x = -1$,
max when $x = 2$

(d) $x = -1, y = -7$;
 $x = 2, y = 20$

(e)



(iii) (a) $\frac{dy}{dx} = 3x^2 - 8x$;
 $x = 0, x = 2\frac{2}{3}$

(b) $\frac{d^2y}{dx^2} = 6x - 8$.

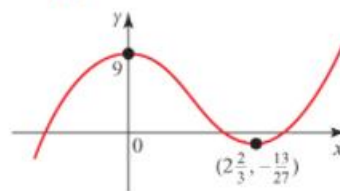
When $x = 0, \frac{d^2y}{dx^2} = -8$

When $x = 2\frac{2}{3}, \frac{d^2y}{dx^2} = 8$

(c) Max when $x = 0$,
min when $x = 2\frac{2}{3}$

(d) $x = 0, y = 9$; $x = 2\frac{2}{3}$,
 $y = -\frac{13}{27}$

(e)



(iv) (a) $\frac{dy}{dx} = 3x^2 - 4x + 1$;
 $x = \frac{1}{3}; x = 1$

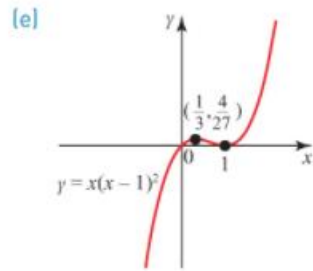
(b) $\frac{d^2y}{dx^2} = 6x - 4$

When $x = \frac{1}{3}, \frac{d^2y}{dx^2} = -2$

When $x = 1, \frac{d^2y}{dx^2} = 2$

(c) Max when $x = \frac{1}{3}$;
min when $x = 1$

(d) $x = \frac{1}{3}, y = \frac{4}{27};$
 $x = 1, y = 0$

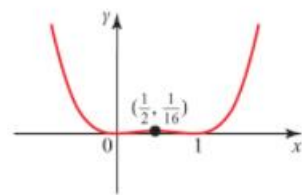


(v) (a) $\frac{dy}{dx} = 4x^3 - 6x^2 + 2x$
 $x = 0, \frac{1}{2}$ and 1

(b) $\frac{d^2y}{dx^2} = 12x^2 - 12x + 2$
 When $x = 0, \frac{d^2y}{dx^2} = 2$
 When $x = \frac{1}{2}, \frac{d^2y}{dx^2} = -1$
 When $x = 1, \frac{d^2y}{dx^2} = 2$

(c) Min when $x = 0$
 Max when $x = \frac{1}{2}$
 Min when $x = 1$

(d) $x = 0, y = 0$
 $x = \frac{1}{2}, y = \frac{1}{16}$
 $x = 1, y = 0$



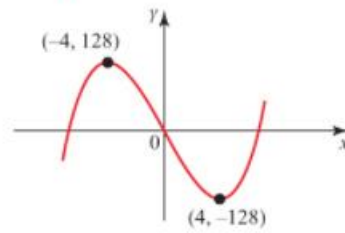
(vi) (a) $\frac{dy}{dx} = 3x^2 - 48$
 $x = -4, x = 4$

(b) $\frac{d^2y}{dx^2} = 6x$
 When $x = -4, \frac{d^2y}{dx^2} = -24$
 When $x = 4, \frac{d^2y}{dx^2} = 24$

(c) Max when $x = -4,$
 min when $x = 4$

(d) $x = -4, y = 128;$
 $x = 4, y = -128$

(e)



(vii) (a) $\frac{dy}{dx} = 3x^2 + 12x - 36$
 $x = -6, x = 2$

(b) $\frac{d^2y}{dx^2} = 6x + 12$

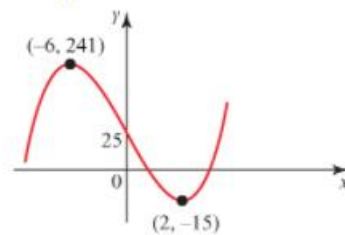
When $x = -6, \frac{d^2y}{dx^2} = -24$

When $x = 2, \frac{d^2y}{dx^2} = 24$

(c) Max when $x = -6,$
 min when $x = 2$

(d) $x = -6, y = 241$
 $x = 2, y = -15$

(e)



(viii) (a) $\frac{dy}{dx} = 6x^2 - 30x + 24$
 $x = 1, x = 4$

(b) $\frac{d^2y}{dx^2} = 12x - 30$

When $x = 1, \frac{d^2y}{dx^2} = -18$

When $x = 4, \frac{d^2y}{dx^2} = 18$

(c) Max when $x = 1,$
 min when $x = 4$

(d) $x = 1, y = 19$
 $x = 4, y = -8$

(e)

