Matrices and Transformations

Exercise 9C

① Work out the 2×2 matrix that represents each of the following transformations.

(i) Reflection in the x-axis.

(ii) Rotation of 90° about O.

(iii) Enlargement, scale factor 2, centre the origin.

(iv) Reflection in the y-axis.

(v) Reflection in the line $y = x$.

(vi) Rotation by 180°, centre the origin.

(vii) Reflection in the line $y = -x$.

(viii) Enlargement, scale factor -3, centre O.

(ix) Enlargement, centre O, scale factor $\frac{1}{2}$.

② The unit square OABC is transformed by the matrix $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ to OA′B′C′.
Show the image on a diagram, labelling each vertex.

③ The unit square OABC is transformed by the matrix $\begin{bmatrix} -\frac{1}{2} & 0 \\ 0 & -\frac{1}{2} \end{bmatrix}$ to OA′B′C′.
Show the image on a diagram, labelling each vertex.

④ Describe fully the transformations given by the following matrices.

(i) $\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$ (ii) $\begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}$ (iii) $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

(iv) $\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$ (v) $\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$ (vi) $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$

(vii) $\begin{bmatrix} \frac{3}{2} & 0 \\ 0 & \frac{3}{2} \end{bmatrix}$ (viii) $\begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$

⑤ The unit square OABC is transformed to OA′B′C′.
OA′B′C′ is shown on the diagram.

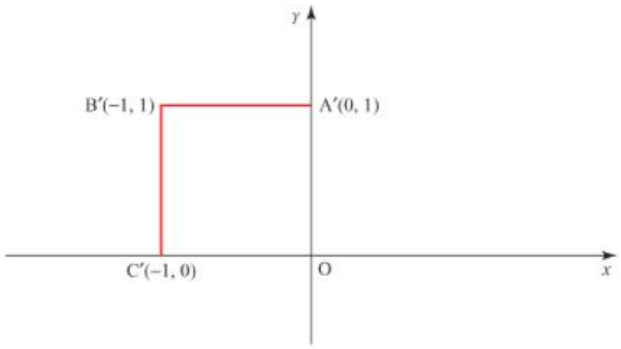

Figure 9.5

Work out the matrix for the transformation.

⑥ The unit square OABC is transformed by the matrix $\begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix}$ to OA′B′C′.

Work out the area of OA′B′C′.

⑦ The unit square OABC is transformed by the matrix $\begin{bmatrix} k & 0 \\ 0 & k \end{bmatrix}$ to OA′B′C′.

The area of OA′B′C′ is 64 square units.

Work out the two possible values of k.

⑧ (i) Draw a diagram to show the unit square OABC rotated 45° about the origin.

(ii) Work out the coordinates of A′ and C′ (the images of A and C).

(Hint: $\sin 45° = \frac{\sqrt{2}}{2}$ and $\cos 45° = \frac{\sqrt{2}}{2}$.)

(iii) Hence write down the transformation matrix for a rotation of 45° about the origin.